ω-regular Energy Problems

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Energy Büchi Problem

- Timed automata
- Büchi condition
- weighted over integers
  - negative weight: consumption of energy
  - positive weight: collection of energy
- energy bounded
  - from below (battery must not become empty)
  - weakly from above (maximal battery capacity)

Energy Büchi Problem

Does a Büchi accepted feasible run exist?

energy always within bound $[0, b]$

weak upper bound b

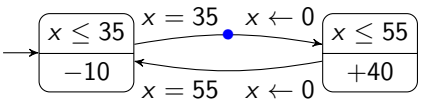
Remember, Remember, the 15 September

- ... 2008
- Bouyer, F., Larsen, Markey, Srba: *Infinite Runs in Weighted Timed Automata with Energy Constraints*, FORMATS 2008
- Dziadek, Fahrenb., Schlehuber: *Energy Büchi Problems*, FM 2023:
  - extend to Büchi conditions
  - fix problems
  - implement everything: TChecker + Spot
- Dziadek, Fahrenb., Schlehuber: *ω-regular Energy Problems*, submitted
  - extend to Parity condition
  - fix more problems
  - add trace extraction
  - update implementation

Weighted Timed Büchi Automata

Weighted Timed Büchi Automata

- generalized Büchi acceptance on transitions
- (only) locations are weighted



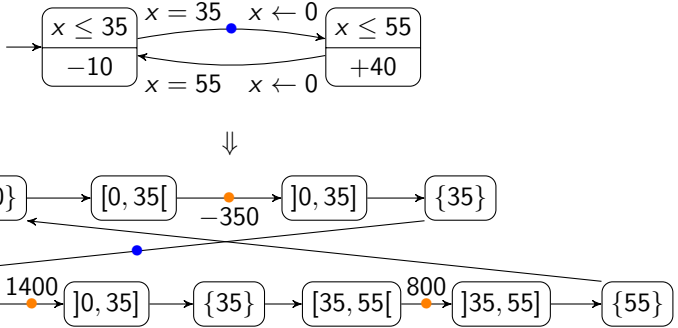
Note: we only handle one clock

Energy problems **undecidable** for **four** clocks (Bouyer, Larsen, Markey 2014)
open for **two** or **three** clocks

Corner-Point Abstraction

One-clock timed automaton → untimed automaton

- TChecker computes the zone graph
- compute corner-point abstraction (Behrmann, Fehnker, Hune et al. 2001)
- Zeno-exclusion



Weighted Büchi Automata

Weighted Büchi Automata

Weights

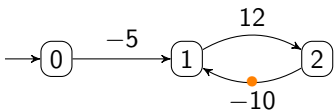
Given values: c : initial credit
b : weak upper bound

Weights:
$$e_0 = \min(b, c)$$
$$e_{i+1} = \min(b,\ e_i + w_i) \qquad \text{for transition weight } w_i$$

Feasible Run

Always: $e_i \geq 0$

Example



Feasible with $c \geq 5$ and $b \geq 10$.

Details: Bellman-Ford & Büchi

Bellman-Ford (BF)

Recall: BF finds **shortest** paths $\Rightarrow$ Invert to find **maximal** energy

BF relaxes a distance approximation until solution is found

BF asserts that no "negative loops" exist $\not\Leftrightarrow$ here, positive cycles are desired

BF not aware of Büchi acceptance

Our solution:

- Decompose **strongly connected components**
- Treat accepting **back-edges** one-by-one
- **modify BF** for "energy positive" loops

Our Algorithm

Take a weighted Büchi automaton:

- find strongly connected components (SCC) (we use Couvreur)
- degeneralize SCCs (produces Büchi accepting **back edges**)
- with modified Bellman-Ford search for feasible lassos:
  - on original graph for maximal **prefix** energy
  - in SCCs for **non-negative cycles** including a **Back-edge**

Note: Energy and Büchi condition cannot be fully separated

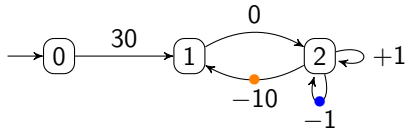


Figure: Original WBA

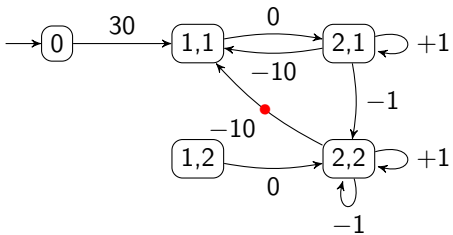
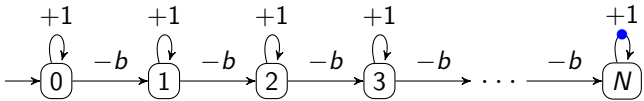


Figure: Degeneralizing SCC $\{1, 2\}$ with level 1 rooted in

Modified BF: Challenges

Example



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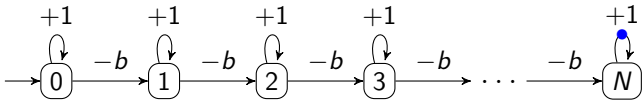

Get weak upper bound b out of complexity

Modified BF: Challenges

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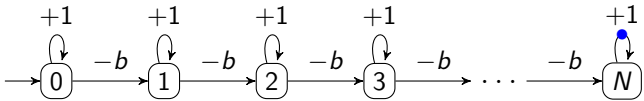

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$\Rightarrow$ After each iteration, positive loop are *pumped*

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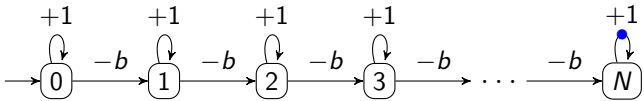

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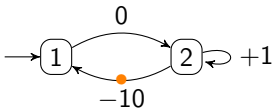
Example (for $c = 30, b = 30$)



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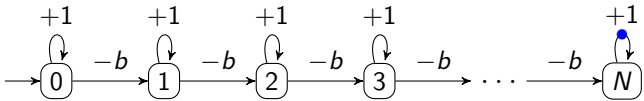

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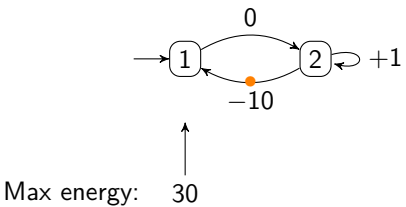

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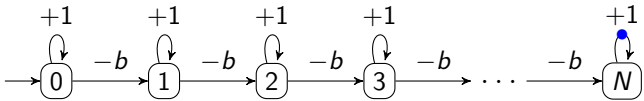

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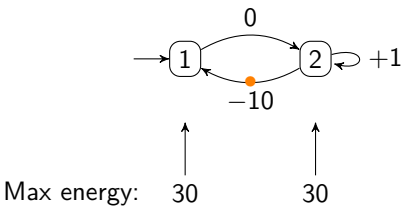

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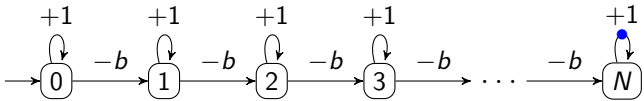

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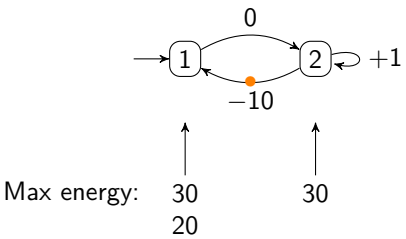

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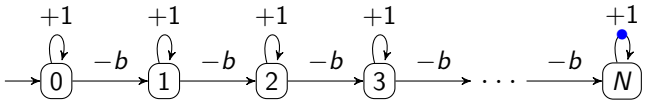

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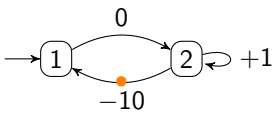
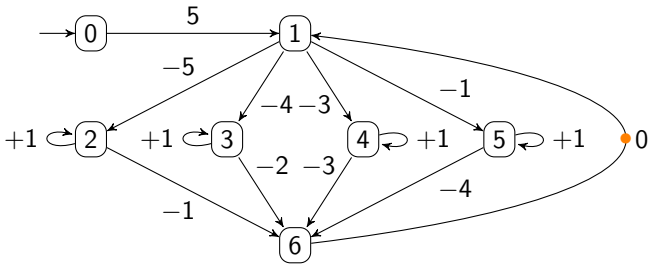
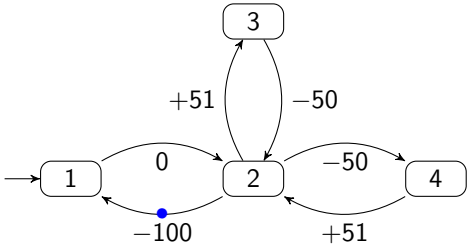


Max energy: 30 30
20

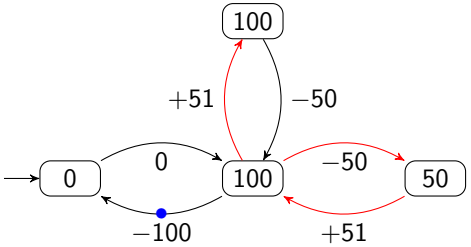


WBA where two iterations do not suffice

Challenges trace extraction

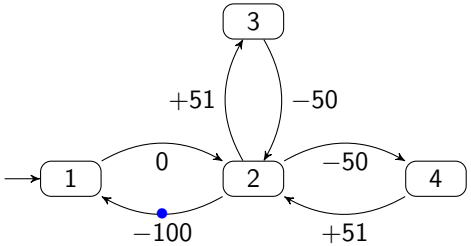


Nontrivial example of trace extraction (for $b = 100$)



Results obtained from the Algorithm.

Challenges trace extraction



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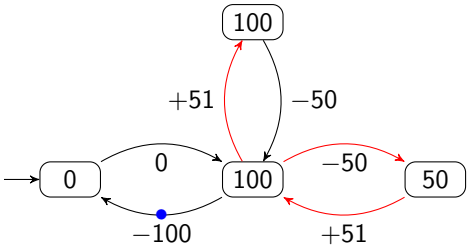
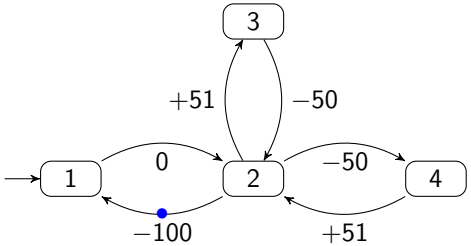


Results obtained from the Algorithm.

Important information is lost during the iterations.

Challenges trace extraction



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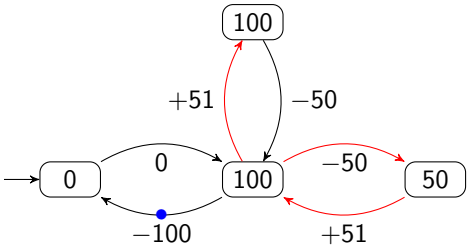
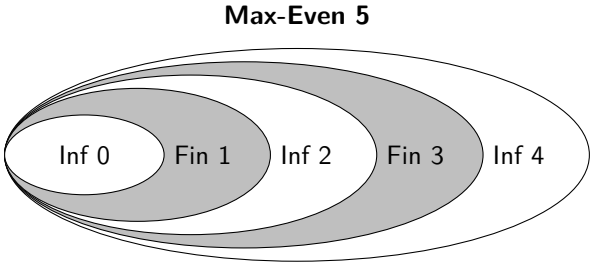


Results obtained from the Algorithm.

Important information is lost during the iterations.

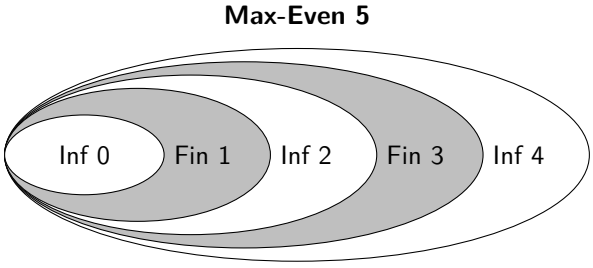
$\Rightarrow$ Storing all predecessors and launch an adapted backwards-forward search.

From Büchi to Parity



Parity condition for priorities up to 4:
$\mathsf{Inf}(4) \mid (\mathsf{Fin}(3) \,\&\, (\mathsf{Inf}(2) \mid (\mathsf{Fin}(1) \,\&\, \mathsf{Inf}(0))))$

From Büchi to Parity



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$\Rightarrow$ Reduce to Büchi case from most to least important color

Conclusion

Results on Energy Büchi problems

1. *Weighted ω-regular automata*
   - Modified Bellman-Ford with Couvreur's algorithm
2. *One-clock weighted timed ω-regular automata*
   - Reduce to 1. using corner-point abstraction
3. *Solved the trace extraction problem*

All algorithms are implemented using TChecker and Spot

Future Work

- edge weights
  - Bouyer, F., Larsen, Markey: *Timed automata with observers under energy constraints*, HSCC 2010
- Avoid iteration over all maximal states.
- parametric problem: synthesize b and/or c
  - F., Juhl, Larsen, Srba: *Energy Games in Multiweighted Automata*, ICTAC 2011
  - (in some cases that's easier!)
- implement everything!